

Sundara

In slide 3 of the Lecture 11, U_1 and U_2 are seen as sub schemes of A^2 . However, are not they (naturally) only a sub scheme of A^3 ?

I understand that we want to "ignore" the data carried by the u_1 and the u_2 coordinates and in some sense project this onto A^2 , but I'm not able to see why this will still be a valid Z affine sub scheme of A^2 (probably it is a quasi affine sub scheme of A^2). An explanation of this will be helpful.

① Closed subscheme of an affine scheme.

$Y \rightarrow X \iff \mathcal{O}(X) \rightarrow \mathcal{O}(Y)$ is onto
 $\swarrow \searrow$
 affine.

② Open subscheme of an affine scheme

$Y \rightarrow X \iff \exists I \text{ s.t. } Y = \mathcal{O}(\mathcal{O}(X), I)$

③ General "subscheme"

$Y \rightarrow X$ morphism of functors

s.t. $Y(R) \rightarrow X(R)$ is 1-1 for all rings R

in some both.
 ④ Alt. in terms of σ $\sigma(Y) \hookrightarrow \sigma(X)$ topological subspace

③ quasi-affine subschemes

$Y \rightarrow X$ open
 $\swarrow \searrow$
 affine.

$X \rightarrow \text{Sp}(\mathcal{O}(X))$
 $\uparrow \nwarrow$
 Y X is affine

$\exists I \subset \mathcal{O}(X)$ ideal s.t. $a: \mathcal{O}(X) \rightarrow T$
 Given $\text{sing } T \times a \in X(T)$

When does $a \in Y(T)$

$a \in Y(I) \iff a(I)T = T$

iff $\{a \in \mathcal{O}(\mathcal{O}(X), I)(T)\}$

$\mathcal{O}(X) \xrightarrow{a} T \subset X(T)$
 s.t. $a(I)T = T$

$X \rightarrow \text{Sp}(\mathcal{O}(X))$

q-affine \implies is open
 is open affine

$\exists Y$ affine s.t.
 $X \hookrightarrow Y$
 s.t. $X = \mathcal{O}(\mathcal{O}(Y), J)$

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If X is a sub scheme of Y , what can we say about the map $O(Y)$ to $O(X)$. Conversely, what conditions do we need on a map $O(Y)$ to $O(X)$ to ensure that X is a sub scheme of Y .

The natural guess is that (with intuition from Algebraic Sets/Varieties) $O(Y)$ to $O(X)$ should be surjective. However, I'm unable to prove that. I tried using the exactness property of the $\text{Hom}(-, T)$ (over Abelian groups) functor, however, still I'm not able to get a necessary and sufficient condition.

Any thoughts/comments will be helpful.

③ $Y \hookrightarrow X$ subfunctor

Doesn't say much about $O(Y)$; effect if X is affine.

$$X = \text{Spec}(\mathbb{Z})$$

$$\text{Sp}(\mathbb{Z})(T) = \text{single } \#T$$

$$\mathbb{Z} \rightarrow T \text{ is unique}$$

$$\text{Sp}(\mathbb{F}_p) \quad \mathbb{Z} \rightarrow \mathbb{F}_p \rightarrow T \quad p \text{ prime}$$

$$\mathbb{Z} \rightarrow T \quad \text{s.t. } p \rightarrow 0$$

Characterizing T

$$\text{Sp}(\mathbb{F}_p) \hookrightarrow \text{Sp}(\mathbb{Z})$$

$$\text{Sp}(\mathbb{F}_p)(T) \neq \emptyset \text{ iff } T \text{ has characteristic } p$$

Subfunctor of $\text{Spec}(\mathbb{Z})$ means some criteria/property rings (commutative)

① When is this a sheaf

② Since \mathbb{Z} is this a sheaf

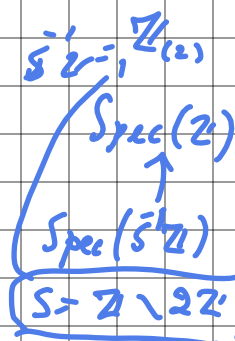
$$\text{Sp}(\mathbb{Z}/\langle N \rangle)$$

$$\mathbb{Z}_N = \mathbb{Z}[u] / \langle u^N - 1 \rangle$$

$$\text{Spec}(\mathbb{Z}_N) \rightarrow \text{Spec}(\mathbb{Z})$$

$$\text{Spec}(\mathbb{Z}_N)(T) \neq \emptyset \text{ iff } N \text{ is a unit in } T.$$

$$0 \notin S \subset \mathbb{Z}, \text{ mult set. } \delta \rightarrow \text{Units} = T$$



Warning: Topology $\rightarrow \sigma$ is the

subfunctor need not be the same topology

$$\sigma(E) \xrightarrow{\sigma} \bigcup_i \sigma \xrightarrow{\sigma} X \xrightarrow{\sigma} Y$$

Topology can form quotient a lot better

$$\mathbb{R} \xrightarrow{f} \mathbb{R}^2$$

$$\mathbb{R} \xrightarrow{g} f(\mathbb{R})$$

$f(\mathbb{R})$ subspace topol
is not
 \mathbb{R}