Sundara

In slide 3 of the Lecture 11, U_1 and U_2 are seen as sub schemes of A^2. However, are not they (naturally) only a sub scheme of A^3?

I understand that we want to "ignore" the data carried by the u_1 and the u_2 coordinates and in some sense project this onto A^2, but I'm not able to see why this will still be a valid Z affine sub scheme of A^2 (probably it is a quasi affine sub scheme of A^2). An explanation of this will be helpful.

(1) Closed subscheme of an affine scheme.

$Y \to X \iff \mathcal{O}(X) \to \mathcal{O}(Y)$ is onto

affine.

(2) Open subscheme of an affine scheme

$Y \to X \iff \exists I$ s.t. $Y = Q(\mathcal{O}(X), I)$

(3) General "subscheme"

$Y \to X$ morphism of functors

s.t. $Y(R) \to X(R)$ is 1-1 for all rings R

in some books [(4) Alt. in terms of σ $\quad \sigma(Y) \hookrightarrow \sigma(X)$ topological subspace

(3') quasi-affine subscheme

$Y \to X$ open

affine.

$X \to Sp(\mathcal{O}(X))$

X is affine

Y

$\exists I \subset \mathcal{O}(X)$ ideal s.t. $a : \mathcal{O}(X) \to T$

Given ring T & $a \in X(T)$

When does $a \in Y(T)$

$a \in Y(T)$ iff $a(I)T = T$

iff $\{a \in Q(\mathcal{O}(X), I)(T)$

$\mathcal{O}(X) \xrightarrow{a} T \quad \subset X(T)\}$

s.t. $a(I)T = T$

$\boxed{X \to Sp(\mathcal{O}(X))}$ is open $\iff \exists Y$ affine s.t.

q-affine — is open

is open affine

$X \hookrightarrow Y$

s.t. $X = Q(\mathcal{O}(Y), J)$

Sundara

If X is a sub scheme of Y, what can we say about the map O(Y) to O(X). Conversely, what conditions do we need on a map O(Y) to O(X) to ensure that X is a sub scheme of Y.

The natural guess is that (with intuition from Algebraic Sets/Varieties) O(Y) to O(X) should be surjective. However, I'm unable to prove that. I tried using the exactness property of of the Hom(-,T) (over Abelian groups) functor, however, still I'm not able to get a necessary and sufficient condition.

Any thoughts/comments will be helpful.

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3 $Y \hookrightarrow X$ subfunctor

Doesn't say much about $\mathcal{O}(Y)$;

even if X is affine.

$X = \mathrm{Spec}(\mathbb{Z})$ $\qquad$ $\mathrm{Sp}(\mathbb{Z})(T) = \text{singleton } \forall T$

$\mathbb{Z} \to T$ is unique

$\mathrm{Sp}(\mathbb{F}_p)$ $\qquad$ $\mathbb{Z} \to \mathbb{F}_p \to T$ $\qquad$ p prime

$\mathbb{Z} \to T$ s.t. $p \mapsto 0$

Characteristic of T

$\mathrm{Sp}(\mathbb{F}_p) \hookrightarrow \mathrm{Sp}(\mathbb{Z})$

$\mathrm{Sp}(\mathbb{F}_p)(T) \neq \phi$ iff T has characteristic p

iff $p = 0$ in T

Subfunctor of $\mathrm{Spec}(\mathbb{Z})$ means

Some criterion/property rings

~~([illegible])~~

1 When is this a sheaf

2 When is this a scheme

$\mathrm{Sp}(\mathbb{Z}/\langle N \rangle)$

$\mathbb{Z}_N = \mathbb{Z}[u]/\langle uN - 1 \rangle$

$\mathrm{Spec}(\mathbb{Z}_N) \to \mathrm{Spec}(\mathbb{Z})$

$\mathrm{Spec}(\mathbb{Z}_N)(T) \neq \phi$ iff.

N is a unit in T.

$0 \notin S \subset \mathbb{Z}$, mult set.

$S \to$ Units in T

$S^{-1}\mathbb{Z} = \mathbb{Z}_{(2)}$

$\mathrm{Spec}(\mathbb{Z})$

$\uparrow$

$\mathrm{Spec}(S^{-1}\mathbb{Z})$

$\boxed{S = \mathbb{Z} \setminus 2\mathbb{Z}}$

Warning: Topology on the (σ)

subfunctor need not be subspace topology

$$\sigma(E) \rightrightarrows \sigma(U') \to X \hookrightarrow \sigma(Y)$$

Topology comes from quotient not sub

$$\begin{tikzcd} \mathbb{R} \arrow[r, "f"] \arrow[dr, "\text{bij}"'] & \mathbb{R}^2 \\ & f(\mathbb{R}) \arrow[u] \end{tikzcd}$$

$f(\mathbb{R})$ subspace topol is not $\mathbb{R}$