Flip Session 10

MTH437 — Introduction to Schemes

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Due to some error the original handwritten file from the session got lost. The following is reproduced to the best of my recollection!

Please explain the remark in Quiz 4 that the $\mathbb{R}_.$ represents the sheaf of continuous functions on X.

Question 10 in Quiz 4 tries to justify that there is a sheaf representing continuous maps to a space Y.

This is based on the following ideas:

- If $U = \cup_i U_i$, then a (set) map $s : U \to Y$ is determined once we are given $s_i = s_{|U_i}$.
- Conversely, given $s_i : U_i \to Y$ that *agree* on the overlaps $U_i \cap U_j$, there is such a set map.
- The notion of continuity of a map is *local*. In other words, s is continuous if and only if each s_i is continuous.

The remark just applies this to the topological space $Y = \mathbb{R}$.

In fact, the above discussion makes it clear that *any* property of functions that is *local* gives rise to a sheaf!

For example:

- Differentiability is local
- Holomorphicity is local

This is one reason why sheaves are useful in differential geometry as well.