Prakash

① Sheaf as a functor

② Sheaf on a topological space

② is a special case of ①

X, top space

T_X : objects are open sets, morphisms are inclusions $i_V : U \to X$

$F \xrightarrow{f} X$ local homeo

$s : V \to F$

$F(V) = \{ s : V \to F \mid f \circ s = id_V \}$

$F : T_X \to \underline{Set}$ is a functor

Sheaf property.

$V = \bigcup V_i$

$$F(V) \to \prod F(V_i) \rightrightarrows \prod_{i<j} F(V_i \cap V_j)$$

$s \leftrightarrow (s_i)$ s.t. $s_i|_{V_i \cap V_j} = s_j|_{V_i \cap V_j}$

$s_i = s|_{V_i}$

$\mathbb{Z}$ - schemes of finite type

$(\mathbb{Z}[x_1, \dots, x_p] / \langle f_1, \dots, f_q \rangle)$ + patching

So $\mathbb{Q}$ does not appear.

"(k - schemes of finite type)"

Abhigyan:- $R \to R_u \to Q(R)$

R a domain $Q(R)$

$u = 2$ $\quad \mathbb{Z} \to \mathbb{Z}_2 \to Q(R) = \mathbb{Q}$

$\mathbb{Z}_2 = \{ \frac{a}{2^n} \mid a \in \mathbb{Z} \}$ $\quad \mathbb{Q} = \{ \frac{a}{b} \mid b \neq 0 \}$

$\frac{a}{2^n} + \frac{b}{2^m} = \frac{c}{2^k}$ $\quad k = \max(m,n)$

$\frac{s}{u^n} = \frac{t}{u^m}$

$u^k(u^m s - u^n t) = 0$

$R_u = \{0\} \iff$ nilpotent $u \neq 0 \in R$ domain

Localization of R in R a domain is a subring of field of fractions.

$\mathbb{Q}$ is not a f.g. $\mathbb{Z}$ ~~module~~ algebra..

is not a quotient of

$\mathbb{Z}[x_1, \dots, x_n] \to \mathbb{Q}$

$\mathrm{Ker}(\mathbb{Z}[x] \to \mathbb{Z}_2) = \langle 2x - 1 \rangle$

"Construction" of quotient

"Sheafification" is quite complication

1. It exists. $\mathbb{Z} \to \mathbb{Z}_2 \to \mathbb{Z}_6 \to \mathbb{Z}_{30}$

$\to \mathbb{Z}_{210} \to \cdots \mathbb{Q}$.