Barnali

Will please elaborate again why Blue color equality of morphism in the attached note hold?

$X \longrightarrow \mathcal{O}(X)$

$X \to \mathbb{A}^1(X) \; \mathcal{O}(X)$

Note that $\mathcal{O}$ is a functor $\mathbb{Z}$-Aff to **CRing**.

We also have the "forgetful functor" **CRing** to **Set** that "forgets" the ring structure. It associates a ring to the underlying set and a ring homomorphism to the underlying set map.

For a $\mathbb{Z}$-Affine scheme X, then we have

Ring

$$\boxed{\mathrm{Mor}(X, A(x;))} = \boxed{\mathrm{Hom}(\mathbb{Z}[x], \mathcal{O}(X))} = \mathcal{O}(X)$$

$\mathcal{O}(A(x;))$ $X \to \mathbb{A}^1$

So $\mathcal{O}$ can be identified with $A(x;)$.!

Set $\quad = \mathbb{A}^1$

R ring w/ 1

$\mathbb{Z} \longrightarrow R$

$1 \longrightarrow 1$

$\mathbb{Z}[x] \to R$

$\mathbb{Z} \quad 1$

$x \longrightarrow a$

$X \xrightarrow{f} \mathbb{A}^1$

$f^{-1}(0) \to 0$

$R, \langle u_1, \dots, u_k \rangle = 1$; $R \xrightarrow{f_i} R_{u_i}$ $R_{u_i} \xrightarrow{g_{i,j}} R_{u_i u_j}$

$a \in R$ $\quad a \longrightarrow a_i = f_i(a)$ $\quad a/1 \longrightarrow a/1$

$(a_i \in R_{u_i})$

$g_{i,j}(a_i) = g_{j,i}(a_j)$

Suppose $a_i = b_i / u_i^n \in R_{u_i}$

$g_{i,j}(a_i) = g_{j,i}(a_j) \ \forall i,j$

Then $\exists !\, a \in R$ s.t. $f_i(a) = a_i$

$R \longrightarrow (g_{i,j}, -g_{j,i}) : \underset{i>j}{R_{u_i} \oplus R_{u_j}} \longrightarrow R_{u_i u_j} \quad i<j$

$(a_i, a_j) \rightarrow g_{i,j}(a_i) - g_{j,i}(a_j)$

$R \longrightarrow \oplus R_{u_j} \longrightarrow \bigoplus_{i<j} R_{u_i u_j} \longrightarrow \oplus R_{\dots}$

$a \longmapsto (f_i(a))$

$(a_i) \longmapsto (g_{i,j}(a_i) - g_{j,i}(a_j))_{i<j}$

Gopal

A Ring

$0 \longrightarrow R \longrightarrow \oplus R_{u_j} \longrightarrow \oplus R_{u_i u_j}$

$(h_i) : A \longrightarrow R_{u_j}$

$g_{i,j}(h_i) = g_{j,i}(h_j)$

$\exists !\, h : A \to R$ s.t. $h_i = f_i \circ h$

$0 \to B \to C \to D$

$0 \to \mathrm{Hom}(A,B) \to \mathrm{Hom}(A,C) \to \mathrm{Hom}(A,D)$

$A \to R$ is a ring homomorphism.

Glued

What is the use of patching or localization?

$X = A(x_1,\dots,x_n; f_1,\dots,f_m)$

$g : X \to \mathbb{A}^1$ $g^{-1}(0)$

$Z_g = A(x_1,\dots,x_n; f_1,\dots,f_m, g) \hookrightarrow X$

Z_g is closed in X

$X_g = A(x_1,\dots,x_n; f_1,\dots,f_m; g)$ "=" $X \setminus Z_g$

it is "open" in X

$= A(x_1,\dots,x_n; f_1,\dots,f_m, ug-1)$

$R = \mathcal{O}(X)$ $\mathcal{O}(X_g) = R_g$ $g_1 \dots g_k$

$R \to R_{u_i}$ $\Longleftrightarrow$ $X_i \hookrightarrow X$ open sets in X

$\langle u_1,\dots,u_k\rangle = 1$ "$\bigcup X_i$" $= X$

Patching in topology

Covering of X by open sets.

$X_i \xrightarrow{h_i} Y$ $\mathcal{O}(X_i \cap X_j) = R_{u_i u_j}$

$h_i|_{X_i \cap X_j} = h_j|_{X_i \cap X_j}$

$\Rightarrow \exists\, h : X \to Y$

Manifolds

M patching open sets $\subset \mathbb{R}^n$

Open subschemes of

X: obtained by patching affine schemes.

Localization gives a collection of open sets which form a basis for the topology.

Kirti

Does patching imply that $u_1, \dots, u_n$ generate unit ideal ?

$$\begin{tikzcd} & & A \arrow[d] \arrow[dr] \arrow[dll, dotted] & \\ & R \arrow[r] & R_{u_i} \arrow[r] & R_{u_i u_j} \end{tikzcd}$$

Sundar:- $(\mathbb{N}, \geq) \xrightarrow[id]{F} (\mathbb{N}, \geq)$

$(\mathbb{N}, \geq) \xrightarrow[G]{id+2} (\mathbb{N}, \geq)$

$F \to G$ $n \mapsto n+2$

$F(n) = n \to n+2$

$G(n) \to F(n)$

$(n+2) \to n$.

Order

$\leq$

S

objects elts

- maps are identity.

$S \cong G$

Categories & Functors $\to$ Logic, TCS

Wikipedia

Set Theory.

nCatlab.org

stacks project

Zermelo-Frankel Axioms

Ordinals, Cardinals