

Barnali

Will please elaborate again why Blue color equality of morphism in the attached note hold?

$$X \rightarrow \mathcal{O}(X)$$

$$X \rightarrow A^1_{\mathbb{Z}}(X) \rightarrow \mathcal{O}(X)$$

Note that \mathcal{O} is a functor \mathbb{Z} -Aff to \mathbf{CRing} .

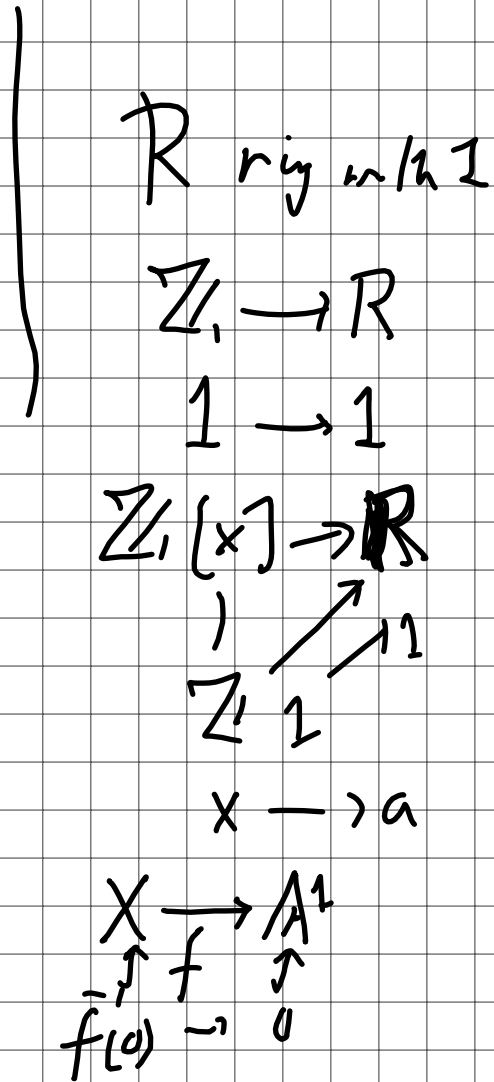
We also have the "forgetful functor" \mathbf{CRing} to \mathbf{Set} that "forgets" the ring structure. It associates a ring to the underlying set and a ring homomorphism to the underlying set map.

For a \mathbb{Z} -Affine scheme X , then we have

$$\boxed{\text{Mor}(X, A(x;))} = \boxed{\text{Hom}(\mathbb{Z}[x], \mathcal{O}(X))} = \mathcal{O}(X)$$

\uparrow Set \uparrow A^1 \uparrow $\mathcal{O}(A(x;))$ \uparrow Ring
 $X \rightarrow A^1$

So \mathcal{O} can be identified with $A(x;)$!



$$R, \langle u_1, \dots, u_n \rangle = 1; \quad R \xrightarrow{f_i} R_{u_i} \quad R_{u_i} \xrightarrow{g_{i,j}} R_{u_i u_j}$$

$$a \in R \quad a \rightarrow a_i = f_i(a) \quad a/1 \rightarrow a/1$$

(a_i \in R_{u_i})

$$g_{i,j}(a_i) = g_{j,i}(a_j)$$

Suppose $a_i = b_i / u_i^n \in R_{u_i}$

$$g_{i,j}(a_i) = g_{j,i}(a_j) \quad \forall i, j$$

Then $\exists! a \in R$ s.t. $f_i(a) = a_i$

$$R \rightarrow (g_{i,j}, -g_{j,i}): R_{u_i} \oplus R_{u_j} \rightarrow R_{u_i u_j} \quad i < j$$

$$(a_i, a_j) \rightarrow g_{i,j}(a_i) - g_{j,i}(a_j)$$

$$R \rightarrow \bigoplus_j R_{u_j} \rightarrow \bigoplus_{i < j} R_{u_i u_j} \rightarrow \dots$$

$$a \mapsto (f_i(a))$$

$$(a_i) \mapsto (g_{i,j}(a_i) - g_{j,i}(a_j))_{i < j}$$

Goal.

$$0 \rightarrow R \xrightarrow{\quad} \bigoplus_j R_{u_j} \xrightarrow{\quad} \bigoplus_{i < j} R_{u_i u_j}$$

$$(h_i): A \rightarrow R_{u_j}$$

$$g_{i,j}(h_i) = g_{j,i}(h_j)$$

$$\exists! h: A \rightarrow R \quad s.t. \quad h_i = f_i \circ h$$

$$0 \rightarrow B \rightarrow (-) \rightarrow D$$

$$0 \rightarrow \text{Hom}(A, B) \rightarrow \text{Hom}(A, C) \rightarrow \text{Hom}(A, D)$$

$A \rightarrow R$ is a ring homomorphism.

Q1 open

What is the use of patching or localization?

$$X = A(x_1, \dots, x_n; f_1, \dots, f_m)$$

$$g: X \rightarrow \mathbb{A}^1 \quad \text{"} \bar{g}(0) \text{"}$$

$$Z_g = A(x_1, \dots, x_n; f_1, \dots, f_m; g) \hookrightarrow X$$

Z_g is closed in X

$$X_g = A(x_1, \dots, x_n; f_1, \dots, f_m; g) = X \setminus Z_g$$

it is "open" in X

$$= A(x_1, \dots, x_n; f_1, \dots, f_m, u_{j-1})$$

$$R = \mathcal{O}(X) \quad \mathcal{O}(X_g) = R_g \quad \underline{g_1 \dots g_n}$$

$$R \rightarrow R_{g_i} \iff X_i \hookrightarrow X \text{ open set}$$

$$\langle u_1, \dots, u_k \rangle = 1 \quad \bigcup X_i = X = X$$

Cover of X by open sets.

$$X_i \xrightarrow{f_i} Y \quad \mathcal{O}(X_i \cap X_j) = R_{u_i u_j}$$

$$h_i|_{X_i \cap X_j} = h_j|_{X_i \cap X_j}$$

$$\Rightarrow \exists h: X \rightarrow Y$$

Patching in topology

Manifolds

M patching open sets

open subscheme of \mathbb{A}^n

X is topology

obtained by patching

affine schemes.

Localization gives a collection of open sets which form a basis for the topology.

Kirti

Does patching imply
that v_1, \dots, v_n generate
unit ideal?



Sundar: $(\mathbb{N}, \geq) \xrightarrow[\text{id}]{F} (\mathbb{N}, \geq)$
 $(\mathbb{N}, \geq) \xrightarrow[\mathbb{G}]{\text{id}+2} (\mathbb{N}, \geq)$

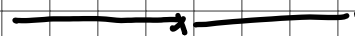
$F \rightarrow G, n \mapsto n+2$
 $F(n) = n \rightarrow n+2$

$G(n) \rightarrow F(n)$
 $(n+2) \rightarrow n$

Order \leq

\sum objects etc
 = maps etc - identity.

$S \ni G$



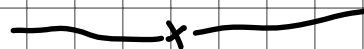
Categories & Functors \rightarrow Logic, TCS

Wikipedia

Set Theory

nCatlab.org

stacks project



{ Zermelo-Frankel Axioms
 Ordinals, Cardinals