~~Prakash~~ Biplob.

Natural transformations

Given functors F and G from $\mathcal{C}$ to $\mathcal{D}$, we have the notion of a *natural transformation* $\eta : F \to G$. (η sends objects of $\mathcal{C}$ to morphisms of $\mathcal{D}$)

This associates, to each object X of $\mathcal{C}$, a morphism $\eta(X) : F(X) \to G(X)$ in $\mathcal{D}$.

Is it arbitrary morphism or induced from $\eta : F \to G$ somehow?

This has the property that if $f : X \to Y$ is a morphism in $\mathcal{C}$, then $\eta(Y) \circ F(f) = G(f) \circ \eta(X)$.

In other words, the following diagram commutes

$$\begin{tikzcd}
F(X) \arrow[r, "\eta(X)"] \arrow[d, "F(f)"'] \arrow[dr, phantom, "\circlearrowright"] & G(X) \arrow[d, "G(f)"] \\
F(Y) \arrow[r, "\eta(Y)"] & G(Y)
\end{tikzcd}$$

$\eta : \mathrm{obj}_{\mathcal{C}} \to \mathrm{mor}_{\mathcal{D}}$

??

$s\eta = F$

$t\eta(x) = F(x)$

$e\eta = G$

$$\begin{tikzcd}
F(X) \arrow[r, "\eta(X)"] \arrow[d, "F(f)"'] \arrow[dr, phantom, "\circlearrowright"] & G(X) \arrow[d, "G(f)"] \\
F(Y) \arrow[r, "\eta(Y)"] & G(Y)
\end{tikzcd}$$

$$\eta(Y) \circ F(f) \;=\; G(f) \circ \eta(X)$$

$f : X \to Y$

$F = id : \mathrm{Vect} \qquad \mathrm{Vect}_k \qquad \mathrm{Hom}(\mathrm{Hom}(V,k),k) = V^*$

$G = \text{double dual} \qquad \mathrm{Hom}(\cancel{V, \mathrm{Hom}}(V,k)) = V^{**}$

V^*

$V \rightleftarrows V^{**}$

$v \mapsto (f \mapsto f(v))$

$V \xrightarrow[f=]{} W \qquad V^{**} \to W^{**}$

Ramanujan Srihari:

Why do we not impose the condition that <a,b> is some non-zero ideal in R? This also seems to capture the idea that not both a and b are 0, but is also milder than demanding <a,b> = R.

$$(\mathbb{A}^2 \setminus \{(0,0)\})(R) = \{ \underset{\in R^2}{(a,b)} \mid \langle a,b\rangle_R = R \}$$

A $\quad R \xrightarrow{f} S$: $\mathbb{Z}_{(3)}$ $\quad \langle 6,9\rangle = \langle 3\rangle \neq 0$

$\langle a,b\rangle \neq \langle 0\rangle \qquad f(\langle a,b\rangle) = \langle 0\rangle$

$\langle a,b\rangle = R \qquad \cancel{f(\langle a,b\rangle) = S}$

$1 = au + bv \qquad \langle f(a), f(b)\rangle_S = S$

$f(1) = f(a)f(u) + f(b)f(b)$

Rishov

Is it true that morphisms need not necessarily be "functions" between the source and target? Can we arbitrarily define morphisms between objects of a category as long as the definition of composition follows the axioms.

I think morphism between two Z-affine schemes is not a "function " as we define the morphism following the ring homomorphism between the corresponding FPR's.

"Small"

Objects form a set

morphism form a set

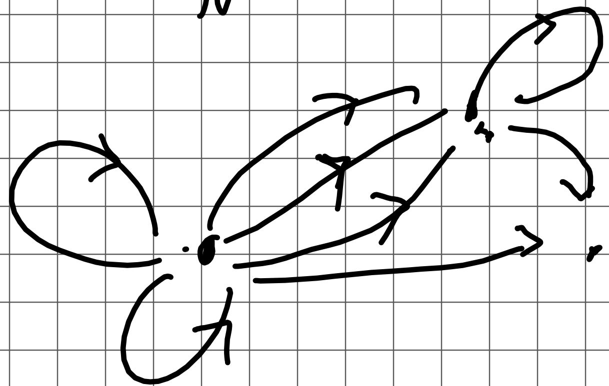
"Set is contradn"

SET

Ring

FPR $\leftrightarrow$ $\mathbb{Z}(x_1,\dots,x_p; f_1,\dots,f_q)$

$\mathbb{Z}$-affine



Vect, k, $k^{\oplus n}$

$(\mathrm{Vect}_{\mathrm{Set}}$, set map$)$

$(\mathrm{Vect}_{\mathbb{C}},)$ fn

$\mathbb{Q}[i] \supset \mathbb{Q}$

$(\mathrm{Vect}_{\mathbb{C}}$, as $\mathbb{R}$-space$)$

$$\begin{array}{ccc} V & \to & V^{**} \\ f \downarrow & & \end{array}$$

Rishov

What are the benefits of treating a Mathematical Structure from categorical viewpoint instead of observing its internal set theoretic structure.

Ease recognition in different contexts.

$\mathrm{Map}(S, k)$ - $\boxed{\text{v. space}}$

Axiomatics → minimizing features

$\mathbb{Z}$-affine scheme-

$R \to X(R)$

$\mathbf{CRing} \longrightarrow \mathbf{Set}$

$X(R) \to Y(R)$

$X = A(x; f)$ $\quad$ $X. \to Y.$

$X \to Y$

$\{X(R)\}_R$ $\quad$ $(X(R), X(R) \to X(S))$

O_{CRing} $\quad$ M_{CRing}

$A(\underline{x}; \underline{f}; \mathbb{Z})$ $\quad$ $\mathbb{Z}$-affine scheme

"pictorial picture"

simultaneous zeros of $f_1, \dots, f_q$ in $\mathbb{A}^p$

$\mathbb{F}_2$