Solutions to Quiz 1

Intersection

Given $\mathbb{P}(V)$ and $\mathbb{P}(W)$ are subspaces of $\mathbb{P}^7$ where V has dimension 4 and W has dimension 5.
What is the *minimum* dimension of their intersection?

(The word "minimum" was missing in the original question.)

Note that V and W are vector subspaces of $k^{7+1} = k^8$. Hence, the dimension of $V \cap W$ is *at least* $4 + 5 - (7 + 1) = 1$.

It follows that the minimum dimension of $\mathbb{P}(V) \cap \mathbb{P}(W) = \mathbb{P}(V \cap W)$ is 0.

Note that the intersection is contained in $\mathbb{P}(V)$ so the dimension of this intersection is *at most* $4 - 1 = 3$. Partial credit was given for answers $1, 2, 3$.

Join of spaces

Given that $\mathbb{P}(V)$ and $\mathbb{P}(W)$ meet in exactly one point in $\mathbb{P}^{10}$, where V has dimension 2 and W has dimension 3.
What is the dimension of $\mathbb{P}(V+W)$?

Given that $\mathbb{P}(V) \cap \mathbb{P}(W) = \mathbb{P}(V \cap W)$ is a point, we see that $V \cap W$ is 1-dimensional.

It follows that $V + W$ has dimension $2 + 3 - 1 = 4$. Hence, $\mathbb{P}(V + W)$ has dimension 3.

Note that dimension of the *ambient* space $\mathbb{P}^{10}$ does not appear in the calculation!

Multiple intersection

Given 4 linear subspaces of dimension 4 in $\mathbb{P}^5$ what is the minimum dimension of their intersection?

A 4-dimensional subspace if $\mathbb{P}^5$ is given by a non-zero linear functional $f : k^{5+1} \to k$. Thus, we are given 4 non-zero linear functionals $f_1, \ldots, f_4$ on k^6.

The dimension of their *common* intersection is *least* if they are linearly independent. In that case, the kernel V of $k^6 \to k^4$ given by the 4-tuple $(f_1, \dot{;} f_4)$ has dimension 2.

It follows that the dimension of $\mathbb{P}(V)$ is 1.

Multiple join

Given 3 points in $\mathbb{P}^3$. What is the maximum possible dimension of their join?

Each point is given by a 1-dimensional subspace in k^{3+1}. Each such subspace is generated by a non-zero vector v. Thus, we are given 3 non-zero vectors v_1, v_2, v_3 in k^4.

The join of these is the projective linear subspace of the vector space V spanned by these. The maximum possible dimension of V is 3.

It follows that the dimension of $\mathbb{P}(V)$ is 2.

Number of points

What is the number of points in the projective space $\mathbb{P}^2(\mathbb{F}_3)$ over the field $\mathbb{F}_3$ with 3 elements?

This is the projective space associated with the vector space $\mathbb{F}_3^{2+1}$ over the field $\mathbb{F}_3$.

This means we need to take non-zero vectors modulo multiplication by non-zero scalars. This action is faithful!

There are $3^3 - 1 = 26$ non-zero vectors and $3 - 1 = 2$ non-zero scalars.

Hence, there are $26/2 = 13$ points in this projective space.