Reconstruction of late time cosmology by Principal Component Analysis (PCA)

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Motivation of Using PCA

- PCA is a non-parametric and model-Independent method.

- It gives the functional form of dependent variable in terms of the independent variable directly from data.

- PCA algorithm along with the Correlation test calculation create a hierarchy of priority which differentiate signal from noise.
Requirement for Reconstruction

Numerical value of $x$ vs $\xi(x)$

Independent variable

Dependent variable
Requirement for Reconstruction

Numerical value of $x$ vs $\xi(x)$

Data should have lesser non-linear component than linear

Independent variable

Dependent variable

Correlation test calculation
Algorithm of Reconstruction

1) Expressing the dependent variable in a polynomial

\[ \xi(x) = \sum_{i=1}^{\infty} b_i f(x)^{(i-1)} \]
Algorithm of Reconstruction

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1) Expressing the dependent variable in a polynomial

\[ \xi(x) = \sum_{i=1}^{N} b_i f(x)^{(i-1)} \]
2) Division of N-dimensional parameter/coefficient space in small patches and use $\chi^2$ to find optimize set of parameter values in those patches.

\[
\chi^2 = \sum_{j=1}^{k} \frac{(\xi(x)_{\text{data}} - \xi(\{b\}, x))^2}{\sigma_j^2}.
\]

- Comes from data-set
- From polynomial expansion
PCA algorithm
PCA algorithm

- pca1
- pca2
Algorithm of Reconstruction

3) Finding of Coefficient-Matrix

\[
Y = \begin{pmatrix}
  b_1^{(1)} & b_2^{(1)} & \cdots & b_n^{(1)} \\
  b_1^{(2)} & b_2^{(2)} & \cdots & b_n^{(2)} \\
  b_1^{(3)} & b_2^{(3)} & \cdots & b_n^{(3)} \\
  \vdots & \vdots & \ddots & \vdots \\
  b_1^{(N)} & b_2^{(N)} & \cdots & b_n^{(N)}
\end{pmatrix}
\]

N → Number of dimension [Calculation of correlation]

n → Number of Patches
Algorithm of Reconstruction

4) Co-variance Matrix of Coefficient Matrix

\[ C = \frac{1}{n} YY^T \]

\[ C = \begin{pmatrix}
\sigma_{b_1}^2 & Cov(b_1, b_2) & Cov(b_1, b_3) \\
Cov(b_2, b_1) & \sigma_{b_2}^2 & Cov(b_2, b_3) \\
Cov(b_3, b_1) & Cov(b_3, b_2) & \sigma_{b_3}^2
\end{pmatrix} \]

- From noise
- From signal
5) Diagonalisation of the co-variance Matrix and finding of Eigenfunctions

\[ U = G \varepsilon \]

- **Final basis function**
- **Initial basis-functions**
  \[ G = (f_1, f_2, \ldots, f_N) \]
- **Eigenvector Matrix**
6) Reduction of dimension and the final form

\[ \xi(x) = \sum_{i=1}^{M} \beta_i u(x)^{(i-1)} \]
Reconstruction of $w(z)$ :: Derived approach

$$H^2(z) = H_0^2 \left[ \Omega_m (1 + z)^3 + \Omega_x e^3 \int_0^z \frac{1 + w(z')}{1+z'} dz' \right]$$

$$w(z) = \frac{3H^2 - 2(1 + z)HH'}{3H_0^2(1 + z)^3\Omega_M - 3H^2}.$$
Reconstruction of $w(z)$ :: Derived Approach

\[ \mu(z) = 5 \log \left( \frac{d_L}{1\text{Mpc}} \right) + 25 \]

\[ d_L(z) = \frac{c}{H_0} (1 + z) \int_0^z \left( \Omega_m (1 + z')^3 + \Omega_x e^3 \int_0^{z'} \frac{(1+w(z'))dz'}{(1+z')} \right)^{-1/2} dz' \]
Results

Reconstruction for simulated LCDM Hz data \( f(z) = 1-a \)

Reconstruction for \( w(z) \) from simulated LCDM Hz data
Results

Reconstruction for simulated LCDM Hz data \( f(z) = a \)

Reconstruction for \( w(z) \) from simulated LCDM Hz data
Results

Reconstruction for simulated LCDM $h(z)$ data \( f(z) = z \)

Reconstruction for \( w(z) \) from simulated LCDM $h(z)$ data
Results

Reconstruction for Hz data  
\[ f(z) = 1 - a \]

Reconstruction for SNIa data  
\[ f(z) = 1 - a \]
Results

Reconstruction for Hz data
\[ f(z) = a \]

Reconstruction for SNIa data
\[ f(z) = a \]
Results

Reconstruction for Hz data
\[ f(z) = z \]

Reconstruction for SNIa data
\[ f(z) = z \]
Results

EoS from derived approach.

$h_0 = 0.784001$  \hspace{1cm}  $f(z) = 1-a$
Results (Rejected)

\[ h_0 = 0.739271 \]
\[ f(z) = a \]
Results (Rejected)

EoS from derived approach.

$h_0 = 0.770216 \quad f(z) = z$
Combination of PCA algorithm and correlation test calculation can be used as a reconstruction tool for different observable quantity which has little non-linear dependencies.
Reconstruction of $w(z) ::$ Direct Approach

Comes from PCA

\[ H^2(z) = H_0^2 \left[ \Omega_m (1 + z)^3 + \Omega_x e^3 \int_0^z \frac{1+ w(z')}{1+z'} \, dz' \right] \]

**Advantage**

No intermediate quantity involve

**Disadvantage**

We cautiously add non-linear component
Reconstruction by direct approach, $w(z) = -1$ and $w(z) = -\tanh(1/z)$
Red patch is for all the reconstruction which is differ by the minimum chi reconstruction by 0.3 unit.
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Red patch is for all the reconstruction which is differ by the minimum chi reconstruction by 0.3 unit.
Results Direct approach (1-a)

Reconstruction by direct approach, $w(z) = -1$ and $w(z) = -\tanh(1/z)$

Red patch is for all the reconstruction which is differ by the minimum chi reconstruction by 0.3 unit.
Reconstruction of $w(z)$ from SNIa data

$f(z) = 1 - a$
Reconstruction of $w(z)$ from SNIa data

$f(z) = a$
Reconstruction of $w(z)$ from SNIa data

$f(z) = z$